

Problems Without Figures

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One might expect to find the following sentiment (with calculators and computers instead of calculating machines and skillful accountants) in a contemporary course on teaching math in this post-Common Core world:

"Every problem in arithmetic calls for two distinct and widely different kinds of work: first, the solution, which involves a comprehension of the conditions of the problem and their relation to one another; second, the operation. First we decide what to do; this requires reasoning. Then we do the work; this is a merely mechanical process, and the more mechanical the better. A calculating machine, too stupid to make a mistake, will do the work more accurately than a skillful accountant. Adding, subtracting, multiplying and dividing do not train the power to reason, but deciding in a given set of conditions which of these operations to use and why, is the feature of arithmetic which requires reasoning."

However, this comes from the introduction to Problems Without Figures, a book from well over a century ago. The need to focus on reasoning and understanding over calculations has long been a back and forth struggle in the teaching of math. In my opinion, they need to go hand in hand, but finding good ways to stretch students' thinking on the reasoning can be challenging. When someone showed me this book, I immediately knew it was right to use with one of my sons to help stretch his thinking. A book without numbers (or almost) is such a good mental exercise. What better way to really push a student to think about the reasoning instead of just guessing at operations and trying to solve things willy nilly.

As I read through it, I found that while I really liked it, the old-fashioned language and assumed experiences, while interesting for a historian, could be a distraction for kids today. If the point is to get to the math and visualize based on the world around you, not recall life of a century ago, then trying to read these could be an issue for some students. The book referred to sidewalks made of wood, harrows in fields, wagon wheels, and how much you might need to feed your horse, all with the assumption that these would be everyday experiences for the reader. Most difficult, there were several problems with experiences so foreign to me that I couldn't visualize what the problem meant. Some of the units in common usage more than a century ago, such as rods for distance and gills for volume, simply don't even exist anymore, and others aren't something the vast majority of students will know, such as bushels.

As a result, before giving this to my sons, I decided to give it a facelift. About half the problems stayed the same. For many others, I simply updated the language. Instead of boys buying pocket knives from merchants and ladies buying bulk goods from grocers, I put in kids buying video games or adults shopping at supermarkets. This was mostly unnecessary, but while I was clarifying these problems, I thought making it relatable made sense. For a few problems, I changed things that went deeper by updating measurement systems or experiences while trying to keep the spirit of the problem. But for about a dozen problems, I decided they simply weren't able to be updated.

For those problems, I decided to try instead to update the type of math we expect people to be able to do. I left plenty of measurement problems, but the original text had very few percentage problems and no problems that dealt with statistics or combinations, all things that are more essential to life a century later. I tried to add problems dealing with taxes, discounts, and the vast array of choices that modern life offers.

I hope my problems work and that this set of problems reflects a mix of old and new, with problems about farms and cities, modern life and old fashioned life. This is only the first 150 problems in the original text (slightly less than half). If you're driven to seek out more, the original text can easily be found and you'll see there are plenty more problems without figures.

In the original text, the author warns that, "A few 'catch problems' are put in to entrap the unwary." Those remain. Some of the questions have red herrings or misleading information. Others cannot be solved without more information or cannot be solved at all. For example, a coin dated "B.C." is clearly a fake. A few can easily be solved, but only if you realize they're tricks. For example, a dog standing on three feet weighs the same as a dog standing on all four. The author warns that students should be given no preparation for this. I don't necessarily agree. I tend to warn my students to watch out for a tricky problem. In either case, dealing with tricky problems is something I believe students should learn to do, whether forewarned or not, and learning that sometimes the solution is easier than you realize is a good lesson as well.

According to the book, the intended audience for the problems are 4th to 8th graders. This still seems about right to me as a teacher, though I think many of them would be useful for review in high school. Students can tackle the problems orally or practice writing them out.

1. I have a secret number. If you know three times plus five times plus seven times the secret number, how do you find the secret number?
2. I have a secret number. If you know how much the secret number with a third of the number added is, how can you find the secret number?
3. To find the number of cubic feet in a box, what must you know and what must you do?
4. I know how much one-half of Malik's money is and how much one-fourth of Ned's money is. How can I find how much money they both have?
5. A boy counted his money and asked the price of a soccer ball, a book and a Lego set; he then knew how much more money he would have to earn before he could get all of them. How did he find out?
6. Helen knows how much a box of candy will cost and how much a certain book will cost. How may she find out how much money she will have left after paying for the book and the candy?
7. If you are told how long a fence post is and how high it is, how can you find how deep it is set in the ground?
8. Marissa has a certain amount of money plus one-fourth as much as Lucy has. If I tell you how much money Marissa has, how can you find how much Lucy has?
9. A farmer bought some sheep. If you know how much he paid and how many sheep he bought, how can you find the price of one sheep?
10. A man wore an old watch from his grandfather that kept the time well, but one day when he went on a trip, he realized that the time on his watch no longer matched the time on his smart phone. It was two hours earlier. What happened? In what direction did he travel?
11. I know the length and width of a room and the number of square feet that a gallon of paint will cover. How can I find how many gallon buckets of paint I'll need to buy to paint the ceiling of that room?
12. If I know how many gallons of water it takes to fill a small kiddie pool; how can I find how many quarts to fill seven kiddie pools?
13. How can I find how many times a bicycle wheel will turn in going three miles?
14. How would you find the distance around a square park if you know how long two-thirds of one side is?

15. Your mom decided to buy a variety of bulk foods to make her own 10oz individual trail mix packets for a series of hikes with your nature group. She chose almonds, granola clusters, and dried apricots, but they each had a different price per pound. If you know the price per pound for each, the number of hikers, and the number of hikes, how will you find the amount she spent overall?

16. A zookeeper fed some baby pandas milk, but each one had only half as much as it wanted. If I tell you how many cups of milk she gave them, how will you find how many pints would have given them all they wanted?

17. If I tell you the width of the pickets and the distance between the pickets used in making a fence around a park, what else must you know and what will you do to find how long one side of the park is?

18. A pile of bricks is six feet high; what else must you know and what will you do to find how many bricks it contains?

19. Diego is thinking of joining a discount store that charges a fifty dollar membership fee. What does he need to consider to know if it will save him money?

20. One tree in an orchard bore three times as many apples as the average of all the other trees. If you are told how many pounds that one tree produced, what else must you know and what will you do to find how many pounds grew on the whole orchard?

21. If you know how many feet long and wide a foundation for a house is to be, what else must you know and what will you do to find how many truck loads of earth will be taken out in making the excavation?

22. If you know how many pickets are in a fence which surrounds a square garden, what two things must you know besides, and what will you do to find how many square feet in the garden?

23. A bike, rider and seat for an extra child rider together weigh a certain number of pounds; the rider weighs nineteen times as much as the child's seat. If you are told what two-thirds of the bike's weight is, how will you find the weight of the child's seat?

24. One pipe is two-thirds as long as another. What part of the longer pipe must be cut off and added to the short one to make them of the same length?

25. If I know how much my dog weighs when standing on four feet, how do I find his weight when standing on three feet?

26. Of the animals in a pet store, two-fifths are birds; the rest are hamsters, gerbils and cats. If you are told how many eyes and wings all the animals have, how can you find the number of birds?
27. In a stable are some men and four times as many horses. Given the grand total number of heads, hands, feet and tails, how will you find the number of men?
28. A boy bought some things at the store and paid with a twenty dollar bill at the checkout. He received in change five different denominations of money (bills and/or coins). What was the amount of his purchase? Find six correct answers.
29. If I tell you how much a week a woman gets paid for her job, how many weeks' vacation she takes each year and how much a month all her expenses amount to, how will you find how much she can save in two and a half years?
30. Two boys walk around a circle in the same direction, one walking twice as fast as the other. When will they be the greatest distance apart and how far apart will they then be? When will they be farthest apart if they walk in opposite directions?
31. Abby had a tank of fish but one-fourth of them died. She sold the rest of them at ten dollars each. She paid back a debt to her sister then decided to use the rest of the money to buy gerbils. If I tell you how many fish she had at first, how much she owed her sister, and how many gerbils she bought, how will you find the price of the gerbils?
32. How do you find the total surface of a block having rectangular faces?
33. A boy buys a video game; if you know how much money he now has and how much he had at first, how will you find the price of the video game?
34. If I tell you the weight of a wagon when loaded with mulch and the weight of the empty wagon, what else must you know and what will you do to find how many bags of mulch were on the wagon?
35. How would you find the weight of the water that evaporates in the process of drying swimsuits and beach towels left in a basket?
36. If you know the weight of a fire fighter when he's wearing all his gear and his weight when he visits the doctor, how can you find the weight of his gear?
37. A boy bought oranges, apples and grapes. The oranges cost twice as much as the grapes. If you know the cost of the apples and the total cost, how will you find the cost of the oranges?

38. If given the number of pencils in a box, the number of boxes on a shelf, and the number of shelves, can you find the total number of pencils?
39. What would be a convenient way to find the combined weight of what you eat and drink at a meal?
40. If you know the number of square feet in a floor and the length of it, how can you find the width?
41. Two lots for houses are of the same size; one is square and the other is four times as long as it is wide. Which will require the more fence to enclose it, and how much more?
42. Markers come in boxes of a dozen. Paint is sold by the ounce. Elsa realized she had too many boxes of markers and asked Trevor if he would swap for some paint. They agreed to trade based on how much each cost. What do you need to know in order to find out how much paint Elsa received.
43. A pole stands erect from the bottom of a pond. If I know the total length, also the depth of the water and the height above the water, how may I find how deep it is set in the earth?
44. If I tell you how many dimes and how many nickels a girl has, how can you find the amount of her money?
45. At the top of a castle tower, there's a flagpole. If you know how many inches high each step from the ground to the top is, how many there are, and how many feet long the pole is, how can you find the distance from the base of the tower to the top of the pole?
46. A man bought several snacks at the store; what must you know and what will you do to find how much change he should receive?
47. I have a bundle of sticks each a foot long that I've been placing flat on the ground to make shapes. How can I find how many separate one-foot squares I can enclose with them? How many rectangles two feet long and a foot wide will the sticks enclose?
48. If I tell you how many centimeters long and how many millimeters wide a book cover is, how will you find how many decimeters it is around the book cover?
49. A man bought a latte at Starbucks and gave the barista a five dollar bill. He received three different coins as change. What are at least three different prices the latte could have been?

50. A family uses a certain number of cups of milk each day; how will you find the number of gallons they use in a week?

51. A boy knows at what rate he can walk and at what rate he can ride on a bicycle, also how far it is to the shop where he left his bicycle for repairs. How can he find how long it will take to walk to the shop and return on his bicycle?

52. If it takes Jack forty-five minutes to climb up the hill and Jill thirty minutes to climb up the hill, then how long will it take each of them to climb the hill if Jack starts ten minutes late and Jill runs back down as soon as she gets to the top?

53. I know how many inches long and wide a carpet tile is, and how many feet long and wide a room is; how do I find how many carpet tiles will cover the floor?

54. On one day the temperature was above zero, the next day below. How can you find the difference in temperature between these two days?

55. If you know the temperature for each day for a number of days, how can you find the average temperature for those days?

56. A dining room table has two leaves of equal size. I know how many inches wide the table is also the length of the top when both leaves are down. If I tell you the length of one leaf, how will you find the number of square inches in the top when both are up?

57. Which is larger, two hundred seven thousandths or two hundred and six thousandths?

58. Knowing the sum of three consecutive numbers, can you find each of the numbers?

59. If you know the cost of a box of bananas, how can you find the profit they will bring when sold at nineteen cents each?

60. If you drink a fraction of a bottle of soda that is ten inches tall, how can you find how many liters of soda you drank?

61. If you know how tall and wide a building is, how will you then find the building's footprint (how much area it takes up on the ground)?

62. If one-third of a mural on the side of a building is painted by an artist, half the remaining surface is painted by students, and the remainder is unfinished, how can I find the area of the side of the building if I know the length and height of the unfinished portion?

63. A company has boxes of cereal that are all the same size, filled with crunchy O's. They know the weight of one box in grams. How can they find the total weight of the crunchy O's in kilograms?
64. A vine grew a certain number of inches in a night; how can we find the average growth per hour?
65. If you weigh the snow that lies on a square foot of ground, how can you find the weight of snow on a square mile of ground?
66. A boy has a fragment of stone of irregular shape. He wanted to know how many cubic inches it contains. Immersing it in a cup brim full of water and allowing the water to overflow into a saucer, he removes the cup and weighs the saucer and the water which it contains. Then pouring out the water he weighs the empty saucer. If he knows the weight of a cubic foot of water, how can he find the volume of the stone?
67. If you are told the total number of feet, eyes, wings and fingers a certain number of children and three times as many chickens have, how can you find how many children there are?
68. If I know my height in feet and inches, how can I find it in meters?
69. A baker has a number of small baskets of the same size, into which he wants to put an equal number of cookies. When all are sold he wants to make a certain profit. How will he find how many cookies to put into each, and at what price per basket to sell them?
70. John is just as many years older than his brother is younger than his sister. If you know the age brother and of his sister how can you find John's age?
71. A group of five kids wants to play a tournament of one on one basketball. Explain a strategy for figuring out how many different first match ups they could make.
72. A boy bought a piece of candy and gave the checkout clerk a dollar bill, receiving in change three coins, each of a different metal. What was the amount of his purchase?
73. Knowing the length, width and height of a room, the size and number of the doors and windows and the width of the base-board, how can you find how many square feet of drywall on the walls and ceiling?
74. What is the easiest way to find the sum of all the numbers printed on a calendar page for January?

75. The perimeter of a rectangular parking lot has the same value as its area. What is it?

76. A coin was found dated 56 B.C. How old was it?

77. Some dancers are standing in a row, at equal distances apart. If you know the length of the line and the number of dancers, how can you find the distance between them?

78. If you know the length of a street and the number of evenly spaced street lights on it, how can you find the distance between the lights?

79. If the same number of boys are standing at equal intervals on the circumference of a circle the length of which is known, how can you find the distance between two boys, measuring on an arc of the circle?

80. A boy who raised some melons told me the number of vines and the number of melons on each vine; how can I find the average number of melons per vine?

81. John is three times as old as Henry; what must you know and what will you do to find how many times as old as Henry John will be any certain number of years later?

82. On the Metro, the price of a ride is a dollar cheaper per trip for those who have a card than a paper ticket. The card costs several dollars. How can you decide whether it is worth it for a tourist to buy a card?

83. What do you need to do in order to figure out if it's more cost effective to buy regular gasoline (87 octane) or premium gasoline (93 octane) for your car? What variables might you need to consider?

84. Max has a fish tank that holds fifteen gallons of water. He wants to buy a new tank that is 24" by 12" by 16" but he isn't sure if it's bigger, smaller, or the same size as his tank. How can he compare the two of them?

85. Given the length of a straight line of fence and the distance between the posts, how can you find the number of posts? Given the number of posts and the distance between them, how can you find the length of the line of fence? Given the length of the fence and the number of posts, how can you find how far apart the posts are?

86. The trees in a section of an arboretum are planted check-row fashion. (That is, they are planted in lines parallel to the side of the section, with the next row also parallel so that the trees make a dot grid of squares if viewed from above.) If I tell you how far apart they are and how far from the boundary the outer trees are, how can you find the size of the land in that section of the arboretum?

87. Lay three toothpicks in the form of an equilateral triangle. Now arrange six to make the same kind of a figure, two on each side. The second triangle is how many times as large as the first?

88. You want to buy a certain product from to shipped to you from online. Some websites offer it for less than others but the cost of shipping varies. How can you find the cheapest overall option?

89. Some men are in a field picking strawberries at certain price per pound picked. How much will it cost the farmer to have the field picked? What must be known and what done to find the answer?

90. If you know the width of an American flag, how can you find the width of one stripe, and of the blue field?

91. Knowing the height, width and length of a room, how can you find the distance from the northeast corner of the ceiling to the southwest corner of the floor?

92. A student read the same number of pages every day in July. Given the number of pages, if he continues reading at the same rate, how can you calculate how much he will have read by the end of November?

93. A beekeeper sold a candlemaker a cake of beeswax at a certain price per pound. It was afterward found that in the center of the cake was a large cavity. If you know the volume of the cavity in cubic inches, and the weight of a cubic inch of beeswax, how would you adjust the transaction fairly, between the beekeeper and the candlemaker?

94. Dylan made \$85,000 last year. If you know the rate of his federal taxes, the rate of his state taxes, and the amount he took in deductions, can you find what his take home pay was? What else might you need to know?

95. Alyssa sees a notebook on sale at Target for \$1.99. She pays at checkout, then picks up a slice of pizza that's also \$1.99 at the kiosk on the way out. The amount she paid added up to more than \$4. What do you need to know to find out how much Alyssa spent in total?

96. In a box of trading cards some are attack cards, some are energy cards, and the others are character cards. If you are told the total number of cards, the sum of the number of attack cards and energy cards, and the sum of the number of character cards and attack cards, how can you find the number of each type of card?

97. Jose is opening a new taco truck. He only offers tacos, but there are a variety of toppings. How can he figure out how many different tacos people can order at his truck?

98. Knowing how many ounces of a wedge of cheese and the price per pound, how can I find the price of the cheese?

99. Zoe reads in an article that last year that 2.8 million Americans graduated from college. But that number seems wrong to her. What might she need to know to find if the number makes sense?

100. How can you find the weight of the water composing the steam which escapes from a gallon of water in boiling a given time?

101. A board has a wide and a narrow end; if you know the width of each end and the length of the board, how can you find the number of square inches of surface on one side of it?

102. A pole stands upright in a pond. If I tell you the total length of it, the fractional part of it that is in the earth, the fractional part in the water, and that one-third of the part above the water is painted blue and the rest of it white, how can you find the length of the part that is in the air?

103. If the diameter of a wheel is known in feet, how can the number of revolutions it makes in any number of miles be found?

104. Mia is offered two different jobs. One pays more per hour and is ten miles closer to her home. The other job includes insurance and a free gym membership. How can you calculate which job will be more financially beneficial? What other information do you need to know? What variables are there?

106. If you know what fraction of his Legos a boy lost and how many he has left, how can you find how many he lost?

107. Knowing the total profits from a chain of stores and the profits from each individual store except for one, how can you find the profits from that store?

108. A cylindrical bucket holds a gallon; its depth equals its diameter. What are the dimensions of the can?

109. If I know the number of rows of trees in a square orchard and the total number of trees, how can I find the number in each row?

110. If one dog can walk three miles in an hour, how far can four dogs walk in the same time?

111. If I know a third of the selling price of some items and half the cost of the materials to make the items, how can I find the profit or the loss?

112. When Ben goes to buy toilet paper, he sees several options. Each is a different price and different amount per package, but the price per hundred sheets is marked with the price on the shelf. However, one brand is discounted thirty percent and the price per hundred sheets is not marked on the discounted price. How can Ben find which brand is the least expensive?

113. How do you find how many blocks will fit in a box whose length, width, and depth are given?

114. A man decided to get new winter pants, boots, coats, hats, and three pairs of gloves for each member of his family. What must you know and do in order to find how many items he bought altogether?

115. The length of a chain in feet and the number of links in it are known; how can you find the length of a link in inches?

116. James has a certain number of Lego minifigs, and Robert has a smaller number, how can you find how many more James has than Robert? How many times more? How many must James give Robert so that each have the same number?

117. A woman wanted to get a new chest to put in a certain spot in her bedroom between a bookcase and the window. What does she need to do before going to Ikea to make sure she doesn't have to return the chest later?

118. A boy left his bicycle at a shop to be repaired. If you know how many miles distant the shop is, how many miles per hour the boy can walk, and how long it takes him to ride the whole distance, how can you find how long it will take him to walk to the shop and ride back?

119. If I tell you how many white sheep and how many black ones in a flock, how can you tell what percent of them are black?

120. A boy carefully weighed a measuring cup; he then filled it with clean pebbles and weighed it again; after pouring in as much water as the cup would hold with the pebbles in it, he weighed it a third time; then he poured out all the water that would run off and weighed it once more. How can he find the weight of the pebbles, the weight of the water that was poured off, and the weight of the water that clung to the wet pebbles?

121. On a certain day ___ centimeters of rain fell; if I tell you how many kilograms a cubic meter of water weighs, how will you find how many grams fell on a square meter?

122. A boy bought a toy figure, a book and some paper. If I tell you the price of each item, how can you find how much change he will receive if he gives the checkout clerk a twenty dollar bill?

123. One man can finish a piece of work in a week; how long will it take three men to do it? One horse can run around a race course in four minutes; how long will it take four horses to do it?

124. I know the number of feet around a square field; how can I find the number of acres in the field?

125. How many shoes do the men of a certain village wear if three percent of them are one-legged and half the others go bare-footed?

126. Peter needs to send several copies of a book to a friend. There are two ways to send it: by weight or by buying a flat rate box. If Peter knows the weight of the book, the weight of the packaging, and the price per ounce for shipping, how can he decide which method will be less expensive?

127. I know the height of a post and the length of its shadow, also the length of the shadow of a tree; how can I find the height of the tree?

128. If I tell you how many bags of cat food my cat ate in February, how can you tell how many pounds he will eat in the first week of March?

129. When Ayesha gets the bill for her supper, how can she calculate the tip if she wants to leave an extra five dollars for good service?

130. A man built a deck with boards of uniform width. If you know the total number of nails used, the number of nails per board, and the width of the boards, how can you find the length of the deck?

131. Given the cost of an apartment building, and the amount paid annually for taxes, for repairs and for insurance, how can you find how much a month the owner must receive in rent to realize a given percent profit each year on his investment?

132. A boy buys lemons and sugar to make lemonade for a lemonade stand. What must you know to find his profit on each cup?

133. A girl was concerned that her cat might be overeating. She wanted to find the weight of the cat, but the cat weighed too little for the bathroom scale, which can only accurately weigh things that are thirty pounds or greater. How can she find the weight of the cat using the bathroom scale?

134. How can you tell how long it is from a certain time in the morning to a certain time in the afternoon?

135. A table is covered with cloth except a border of wood of uniform width. If you know the number of inches in the length and in the width of the table, also the width of the border in inches, how can you find how many square feet are covered with cloth?

136. A man sold some horses, cows, and sheep, and with the money bought hogs. If you know the number of horses, of cows, and of sheep, also the price of one of each kind of animals, how can you find how many hogs he bought?

137. If you know the width of a piece of wrapping paper in inches, the width and length of each piece used to wrap three identical gifts, and the length of the paper in feet, how can you find if there's enough paper left to wrap one more gift?

138. If July begins on Sunday and you are told how much a day a babysitter receives, how can you find her earnings for July if she worked every weekday?

139. A family owns a large, rectangular meadow. If you know how wide the rows the lawn mower makes when they mow the meadow, then how can you find how far the owner drove the riding mower?

140. I know how many feet long a square city block is and the width of the street in feet; how can I find the number of square yards of pavement required to pave the street around the block?

141. A landscaper bought enough pebbles to fill a bin. If you know the price per ton, the number of tons the bin holds and the number of tons he uses the first month, how can you find the value of the pebbles that are left?

142. A man bought a piece of fabric to make a costume for Halloween. If I tell you the price per yard, the number of yards and the width of the fabric in inches, how can you find the cost per square foot?

143. A store sold a computer for a certain price. If you know how much of the price received was profit, how can you find the price charged?

144. A farmer sold a load of cabbages. If you know the price per head and the average weight per head, also the total amount received for the load, what will you do to find the price per ton?

145. I had a certain sum of money in the bank. After writing several checks for different amounts, how can I find how much remains in the bank?

146. Some phones and tablets are in a store. The value of a phone is less than that of a tablet, but the total value of the phones exceeds that of the tablets. Which is greater, the number of phones or of tablets? How do you know?

147. To discourage overuse of water, a city levied an extra fee of one cent per ten gallons of tap water over 6,000 gallons. If given the amount of a family's water bill and the cost of water per 100 gallons, how can you find how much water they used?

148. Lewis chiseled a square hole through a cubical block; if he should tell you how many inches long the edge of the block is, also the edge of the hole, how could you find the number of cubic inches of wood that remain?

149. If you know the seating capacity of a movie theater, the number of vacant seats, the regular price of admission, and how many children at half price were admitted, how can you find the total amount the theater charged moviegoers for that showing?

150. The upper part of a pole is painted white, the middle part green, and the lower part yellow. If I tell you the height of the pole in feet, how many yards of it are green and how many inches yellow, how will you find how many feet are blue?